Some useful probability distributions:

(i) The Gaussian or normal distribution:

Describes a continuous real random variable x with prob.

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\lambda)^2}{2\sigma^2}\right]$$

$\lambda \to$ mean
$\sigma^2 \to$ dispersion

$\langle x \rangle = \lambda$,

$\langle x^2 \rangle = \sigma^2 + \lambda^2$,

$\langle x^3 \rangle = 3\sigma^2\lambda + \lambda^3$,

$\langle x^4 \rangle = 3\sigma^4 + 6\sigma^2\lambda^2 + \lambda^4$,

...

where $\langle \, \rangle$: (angular brackets) denotes average.

(ii) The binomial distribution: — We have already discussed this in previous lecture notes. Here we review it in concise from

Let ~~...~~ we have a random variable with two outcomes 1 & 2. Example — A coin toss

Relative probabilities — p_1 and $p_2 = 1 - p_1$

Let total no. of trial is — N

1 occurs in n_1 times ,

Then $p_1(n_1) = \binom{N}{n_1} p_1^{n_1} p_2^{N-n_1}$,

where

$$\binom{N}{n_1} = \frac{N!}{n_1!\,(N-n_1)!}$$

Again averages $\rightarrow$

$$\langle n_1 \rangle = Np_1$$

$$\langle n_1^2 \rangle = Np_1 p_2$$

~~For~~ We can generalize binominal distribution for multinomial distribution, ~~other~~ when several outcomes ~~occurs~~ $\{1, 2, 3, \cdots, M\}$ occurs with probabilities $\{p_1, p_2, p_3 \cdots p_M\}$. The probability of finding outcomes $\{n_1, n_2, \cdots, n_M\}$ in a total $N = n_1 + n_2 + \cdots + n_M$ trials is

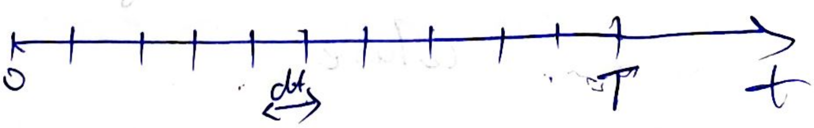
$$\boxed{p_N(\{n_1, n_2, \cdots, n_M\}) = \frac{N!}{n_1!\, n_2! \cdots n_M!}\, p_1^{n_1} p_2^{n_2} \cdots p_M^{n_M}}$$

(iii) The Poission distribution:— Poisson process example is observed in radioactive decay.

Let a piece of radioactive material over a time interval T shows that

(a) prob. of one and only one event in the interval $[t, t+dt]$ is proportional to dt as $dt \to 0$

(b) Probabilities of events at different interval times are independent of each other



The prob. of observing exactly M decays in the interval T is given by Poisson distribution.

Let the prob. of an event occur in each segment is ~~αdt~~ $p = \alpha dt$ and no event occur in each segment $q = 1 - \alpha dt$

The interval is divided into ~~se~~ segments of size dt where

$$\frac{T}{dt} >> 1.$$

Prob. of M events (decay)

$$\boxed{p_{\alpha T}(M) = \frac{e^{-\alpha T} (\alpha T)^M}{M!}}$$

and

$$\langle M \rangle = \alpha T$$

$$\langle M^2 \rangle = (\alpha T)^2 + \alpha T$$

$$\langle M^3 \rangle = (\alpha T)^3 + 3(\alpha T)^2 + (\alpha T).$$